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Forecasting Methodology Based on Alternative Presentation of the Gutenberg–Richter Relation

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Abstract—The main purpose of this study is to propose an innovative methodology to forecast the cumulative probability of future larger earthquakes for any given magnitude. It is based on applying an innovative approach to explicitly incorporate the logarithmic mean annual seismicity rate and its standard deviation, replacing the conventional Gutenberg–Richter (G–R) relation, which is only expressed by the arithmetic mean. The new representation of the G–R relation can provide the median annual seismicity rate and upper and lower bounds of recurrence time period for future larger earthquakes in different regions of Taiwan. Subsequently, the logarithmic mean is found to have a more well-behaved lognormal distribution. The selected crustal earthquake data for $3.0 \le M_{\mathrm{w}} \le 5.0$ are used to obtain alternative Gutenberg–Richter relations for different regions. The results are as follows: $\log_{10} N = 5.74 - 1.07M_{\mathrm{w}} \pm (-0.18 + 0.12M_{\mathrm{w}})$ in and Taiwan; $\log_{10} N = 5.08 - 1.07M_{\mathrm{w}} \pm (0.23 + 0.05M_{\mathrm{w}})$ for northeastern Taiwan offshore; $\log_{10} N = 5.48 - 0.95M_{\mathrm{w}} \pm (-0.32 + 0.14M_{\mathrm{w}})$ for eastern Taiwan offshore; $\log_{10} N = 4.57 - 0.84M_{\mathrm{w}} \pm (0.07 + 0.07M_{\mathrm{w}})$ for southeastern Taiwan offshore. These results can be used for preventing and mitigating seismic hazards.

1. Introduction

Because we do not know how much a block can be loaded and the stresses are from all directions, we rely only on physical models to provide viewpoints of the physics and kinetics of earthquake processes. Taiwan is located at the circum-Pacific seismic belt and between two colliding plates (the Philippine Sea plate on its east side and the Eurasian Plate on its west side). Therefore, earthquakes occur very frequently both on land and in the surrounding offshore areas, especially along the east coast, as shown in Fig. 1; the dataset used is based on a homogenized Taiwan catalog using M_{w} (Chen and Tsai 2008; Chang et al. 2016). However, from records of historical earthquake data, there have been many major earthquakes that have caused catastrophic damage and disastrous loss of life and property in western Taiwan, including the 1906 M_w 7.1 Meishan earthquake, the 1935 M_w 7.1 Hsinchu–Taichung earthquake, and the 1999 M_w 7.6 Chi–Chi earthquake (Hsu 1971; Tsai 1985; Wang and Kuo 1995; Cheng and Yeh 1999; Ma et al. 1999; Shin and Teng 2001). Therefore, it is a critical issue to adopt a reasonable and reliable method to predict the recurrence period for future large earthquakes to prevent and mitigate earthquake disasters in Taiwan.

Gutenberg and Richter (1941, 1944) were two pioneers of modern seismology; each contributed greatly to the development of the field as a modern quantitative science. They developed an empirical relation between the magnitude (M) and the cumulative earthquake number $N(M)$, called the Gutenberg–Richter (G–R) law, which is expressed as follows:

$$\log_{10} N = a - bM \tag{1}$$

where a and b are constants and are determined by a simple least-squares regression, and N is the number of earthquakes with magnitude greater than or equal to M. The N-value, used in the conventional G–R relation, is based on the arithmetic mean. The a-value simply indicates the seismic rate of the region while the b-value describes the seismic activity of the region. This relationship is surprisingly robust and

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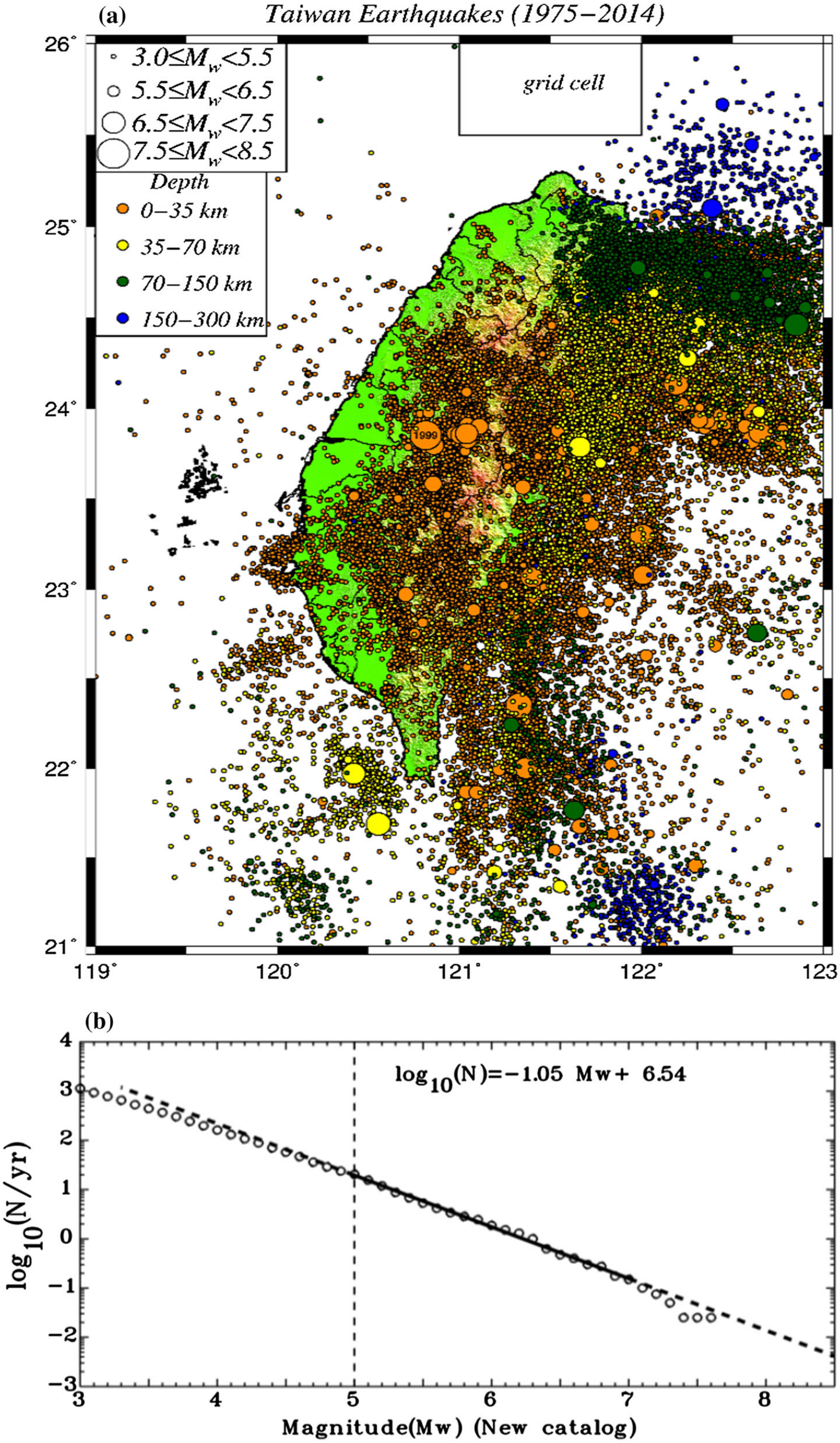

(a)
Taiwan Earthquakes (1975−2014)
3.0≤Mw<5.5
5.5≤Mw<6.5
6.5≤Mw<7.5
7.5≤Mw<8.5
Depth
0−35 km
35−70 km
70−150 km
150−300 km
grid cell
1999
26°
25°
24°
23°
22°
21°
119°
120°
121°
122°
123°
(b)
log10(N)=−1.05 Mw+ 6.54
log10(N/yr)
4
3
2
1
0
−1
−2
−3
3
4
5
6
7
8
Magnitude(Mw) (New catalog)

◀Figure 1

a The epicenter distribution of earthquakes in the Taiwan region with magnitudes $M_w \geq 3.0$ from 1975 to 2014 (Chen and Tsai 2008; Chang et al. 2016). The largest circle labeled with 1999 marks the epicenter of the 1999 $M_w = 7.6$ Chi–Chi earthquake, which also indicates the range of a grid cell. **b** Determination of completeness of the data set. From the figure, we can see it displays a straight line from M_w 3.0 to M_w 7.0; therefore, it is complete and increases the confidence of forecasting

does not vary significantly from region to region or over time. Additionally, large and small earthquakes are found to closely follow the G–R relation. However, the arithmetic mean method is susceptible to excessive influence by anomalously high or anomalously low event numbers. The *a* and *b* values can also indicate the global seismicity (Gutenberg and Richter 1941), and the regional seismicity in southern California (Gutenberg and Richter 1944). Therefore, the Gutenberg–Richter relation provides researchers with a convenient means to conduct quantitative studies of various aspects of seismicity.

In the past, many researchers have been devoted to proposing advanced modified forms of the G–R relation. This includes Aki (1965), who used the maximum likelihood method to estimate the *a*and *b* values and standard deviation, and Wyss (1973), who adopted the seismic moment instead of the magnitude. Kijko (1985) calculated the parameters of a double exponential magnitude-frequency relation using the maximum likelihood method. Ogata and Zhuang (2006) used the G–R relation to study spatial–temporal seismicity variations for earthquake forecasting. Bayrak and Bayrak (2012) quantitatively identified the seismicity properties of any region with the Gutenberg–Richter relation. However, the conventional G–R relation based on the arithmetic mean has the most serious fundamental shortcoming, the arithmetic standard deviation is asymmetric with the arithmetic mean in the log-linear coordinate, it cannot be explicitly incorporated in the G–R relation.

In this study, we propose an innovative approach to explicitly incorporate the logarithmic mean annual seismicity rate and its standard deviation. Since the logarithmic standard deviation is symmetric with respect to the logarithmic mean for a given M_w, after processing the logarithmic mean and the logarithmic standard deviation, we can obtain the regressive equations of the mean, upper, and lower bounds of the logarithmic annual seismicity rate. Finally, the three equations can be integrated into a single equation to express the Gutenberg–Richter relation.

In mathematical terms, the logarithmic mean is a type of mean or average, which indicates the central tendency of a set of numbers by using the product of their values, as opposed to the arithmetic mean, which uses their sum. The logarithmic mean only applies to positive numbers. It is often used for sets of numbers whose values are meant to be exponential in nature, such as an earthquake catalog compiled over a duration of time, globally or in a particular region.

In this study, we propose a novel forecasting methodology (1) transferring the alternative Gutenberg–Richter relation to obtain a corresponding relation for the logarithmic mean recurrence interval and its standard deviation, and (2) using the origin time of the most recent earthquake with a given magnitude, as of a chosen date, to start the cumulative probability curve for that magnitude, to forecast probability of Taiwan future larger earthquake for on Taiwan land, northeastern Taiwan offshore, eastern Taiwan offshore, and southeastern Taiwan offshore, by making use of the characteristics of our innovative processing for Gutenberg–Richter relation that can provide the lower bound, median, and upper bound recurrence time, and logarithmic mean has a more well-behaved lognormal distribution, which can provide the cumulative probability for any given magnitude and recurrence time. Therefore, the results can be of benefit for preventing and mitigating seismic hazards.

2. Data Processing Method

Means are mathematical formulations used to characterize the central tendency of a set of numbers. A geometric mean tends to dampen the effect of very high or low values, which might bias the mean if an average (arithmetic mean) were calculated. The geometric mean is a log-transformation of data to enable meaningful statistical evaluations.

The geometric mean is defined as the nth root of the product of n numbers. For a set of numbers $\{x_1, x_2, x_3, \ldots, x_N\}$, the geometric mean is defined as:

$$\bar{x} = \left(\prod_{i=1}^{N} x_i\right)^{1/N}, \tag{2}$$

where $\bar{x}$ is the geometric mean, x_i is the individual number, and N is the total numbers of the dataset. In practical terms, in the logarithmic domain this equation becomes

$$\log \bar{x} = \frac{1}{N}\sum_{i=1}^{N} \log x_i. \tag{3}$$

In the following, we use the innovative approach of the logarithmic mean to represent the mean annual seismicity rate, together with its standard deviation, for Taiwanese earthquakes with magnitude $M_w \geq 3.0$ in different regions for inland Taiwan, northeastern Taiwan offshore, eastern Taiwan offshore, and southeastern Taiwan offshore. The least-squares fit is also applied to show that the observed annual seismicity rates can be represented closely with a lognormal distribution. Finally, we propose a new representation of the Gutenberg–Richter relation to explicitly incorporate the logarithmic mean and its standard deviation, replacing only the arithmetic mean in the conventional G–R relation.

3. *Lognormal Distributions of the Annual Seismicity Rates*

Taiwan is located between two colliding plates and earthquakes occur very frequently, especially along the eastern coast and the eastern offshore region. Taiwan acts as a sort of earthquake experimental field; therefore, there are ample earthquake datasets, which allow us to have a complete catalog of earthquakes, as shown in Fig. 1. The datasets used are based on the results of Chen and Tsai (2008). Next, we process the observed logarithmic seismicity rates in different regions for inland Taiwan, northeastern Taiwan offshore ($N24.5° - 26.0°, E121.84° - 123.0°$), eastern Taiwan offshore ($N23.0° - 24.5°$, $E121.5° - 123.0°$), and southeastern Taiwan offshore ($N21.0° - 23.0°, E120.9° - 123.0°$) with a lognormal distribution. For this purpose, we aggregate the observed logarithmic seismicity rates in equal bins of 0.05 for Taiwanese earthquakes with magnitude $M_w \geq 3.0$ (the dataset is complete as shown in Fig. 1b, from which we can see it displays a straight line from M_w 3.0 to M_w 7.0; therefore, it is complete and increases the confidence of forecasting for the study) at focal depth 0–35 km (considered confidence at a determined focal depth from 1975) from 1975 to 2014. We then apply a least-squares fit to the observed data using the following equations:

$$Y(x_i) = ay(x_i), \tag{4}$$

$$a = \frac{\sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} y(x_i)}, \tag{5}$$

where $y(x_i) = e^{-\frac{(x-\mu)^2}{2\delta^2}}$, $\mu =$ mean, and $\delta =$ standard deviation, Y_i represents the observed values and $R^2 = \sum_{i=1}^{N} [Y_i - ay(x_i)]^2$, where R is the root-mean-square (RMS) error.

After processing the earthquake data sets with $M_w \geq 3.0$ for inland Taiwan, northeastern Taiwan offshore, eastern Taiwan offshore, and southeastern Taiwan offshore at focal depth 0-35 km, the results are shown in Fig. 2. From Fig. 2a–d we see that the peaks of the lognormal distribution curves are centered in the bulk. The data population with a root-mean-square error is 0.66 for inland Taiwan, the population with an RMS error of 0.67 at northeastern Taiwan offshore, the population with an RMS error of 0.66 at eastern Taiwan offshore, and the population with an RMS error 0.65 at southeastern Taiwan offshore. These phenomena indicate that the observed logarithmic seismicity rates can be fitted closely by a lognormal distribution. This characteristic can provide to forecast the cumulative probability of Taiwan future larger earthquake for any given magnitude.

4. *Explicit Expression of the Gutenberg–Richter Relation in Terms of the Logarithmic Mean Annual Seismicity Rate and Its Standard Deviation*

The Gutenberg–Richter relation is conventionally expressed only by the arithmetic mean annual seismicity rate. The arithmetic mean method has a very

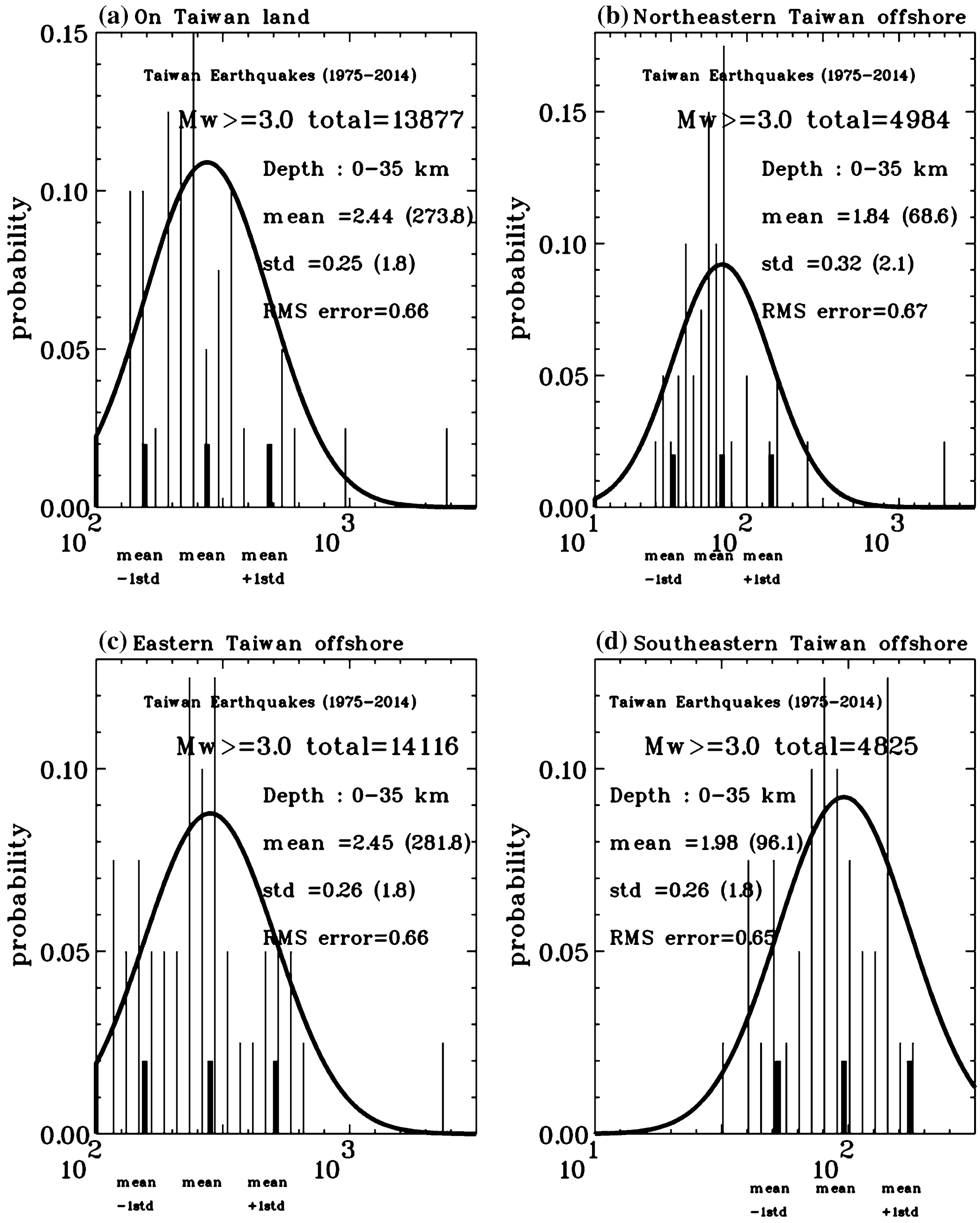


Figure 2
Least-squares fitting by a lognormal distribution with the annual seismicity rates of earthquakes with magnitude $M_w \geq 3.0$ in the Taiwan region from 1975 to 2014 at focal depths 0–35 km: **a** inland Taiwan, **b** northeastern Taiwan offshore, **c** eastern Taiwan offshore, and **d** southeastern Taiwan offshore

serious shortcoming, described as above. In this study, in order to overcome the shortcoming of the arithmetic mean, we propose an innovative approach to express the G–R relation in terms of the logarithmic mean annual seismicity rate and the logarithmic standard deviation. The relation is then used to obtain the corresponding median annual seismicity and the upper and lower bounds for

Taiwanese crustal earthquakes (focal depth 0–35 km) in different regions: inland Taiwan, northeastern Taiwan offshore ($N24.5^\circ - 26.0^\circ, E121.84^\circ - 123.0^\circ$), eastern Taiwan offshore ($N23.0^\circ - 24.5^\circ, E121.5^\circ - 123.0^\circ$), and southeastern Taiwan offshore ($N21.0^\circ - 23.0^\circ, E120.9^\circ - 123.0^\circ$).

For this study, a dataset is selected to cover the period from 1975 to 2014 with a reliable focal depth determination from the Taiwan earthquake catalog with uniform and homogenized moment magnitudes. The seismicity rates in individual regions are used to calculate the logarithmic mean annual seismicity rate and its standard deviation for $3.0 \le M_w \le 5.0$ at an increment of $\Delta M_w = 0.1$ for the focal depth of 0–35 km for inland Taiwan, as well as for northeastern Taiwan offshore, eastern Taiwan offshore, and southeastern Taiwan offshore. The results are shown in Fig. 3a–d, respectively.

We also plot the arithmetic mean annual seismicity rate with the open circle symbols as a comparison. From these figures, we can see that the corresponding standard deviation of each arithmetic mean is asymmetric, as shown by the dashed lines. Therefore, the arithmetic mean annual seismicity rates do not explicitly incorporate the standard deviations in the G–R relation. As stated, we develop an innovative approach to explicitly incorporate the logarithmic mean annual seismicity rate and its standard deviation to be represented in the Gutenberg–Richter relation. The results obtained by best-fitting regression are given below:

For inland Taiwan, as shown in Fig. 3a:

$$\begin{aligned}
&\text{Mean} + \text{std} \quad \log_{10} N = 5.57 - 0.95M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.998 \\
&\text{Mean} \quad \log_{10} N = 5.74 - 1.07M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.999 \\
&\text{Mean} - \text{std} \, \log_{10} N = 5.92 - 1.19M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.999 \\
&\text{Combined} \quad \log_{10} N = 5.74 - 1.07M_w \\
&\quad \pm (-0.18 + 0.12M_w)
\end{aligned} \tag{6}$$

For northeastern Taiwan offshore, as shown in Fig. 3b:

Figure 3 ▶

Plots of the Gutenberg–Richter relation in terms of the logarithmic annual seismicity rate for earthquakes in the Taiwan region from 1975 to 2014 with $M_w \ge 3.0$. The solid dots represent the observed logarithmic mean. The solid vertical bars represent the range of observed logarithmic mean ± one logarithmic standard deviation. The open circle symbols represent the observed arithmetic mean. The dashed vertical bars represent the range of observed arithmetic mean ± one arithmetic standard deviation. The solid line represents the best-fitting regression line constrained by the observed logarithmic mean data, whereas the dotted line represents its extrapolation: **a** inland Taiwanese, **b** northeastern Taiwan offshore, **c** eastern Taiwan offshore, **d** southeastern Taiwan offshore

$$\begin{aligned}
&\text{Mean} + \text{std} \, \log_{10} N = 5.31 - 1.02M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.998 \\
&\text{Mean} \quad \log_{10} N = 5.08 - 1.07M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.999 \\
&\text{Mean} - \text{std} \quad \log_{10} N = 4.86 - 1.12M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.998 \\
&\text{Combined} \quad \log_{10} N = 5.08 - 1.07M_w \\
&\quad \pm (0.23 + 0.05M_w)
\end{aligned} \tag{7}$$

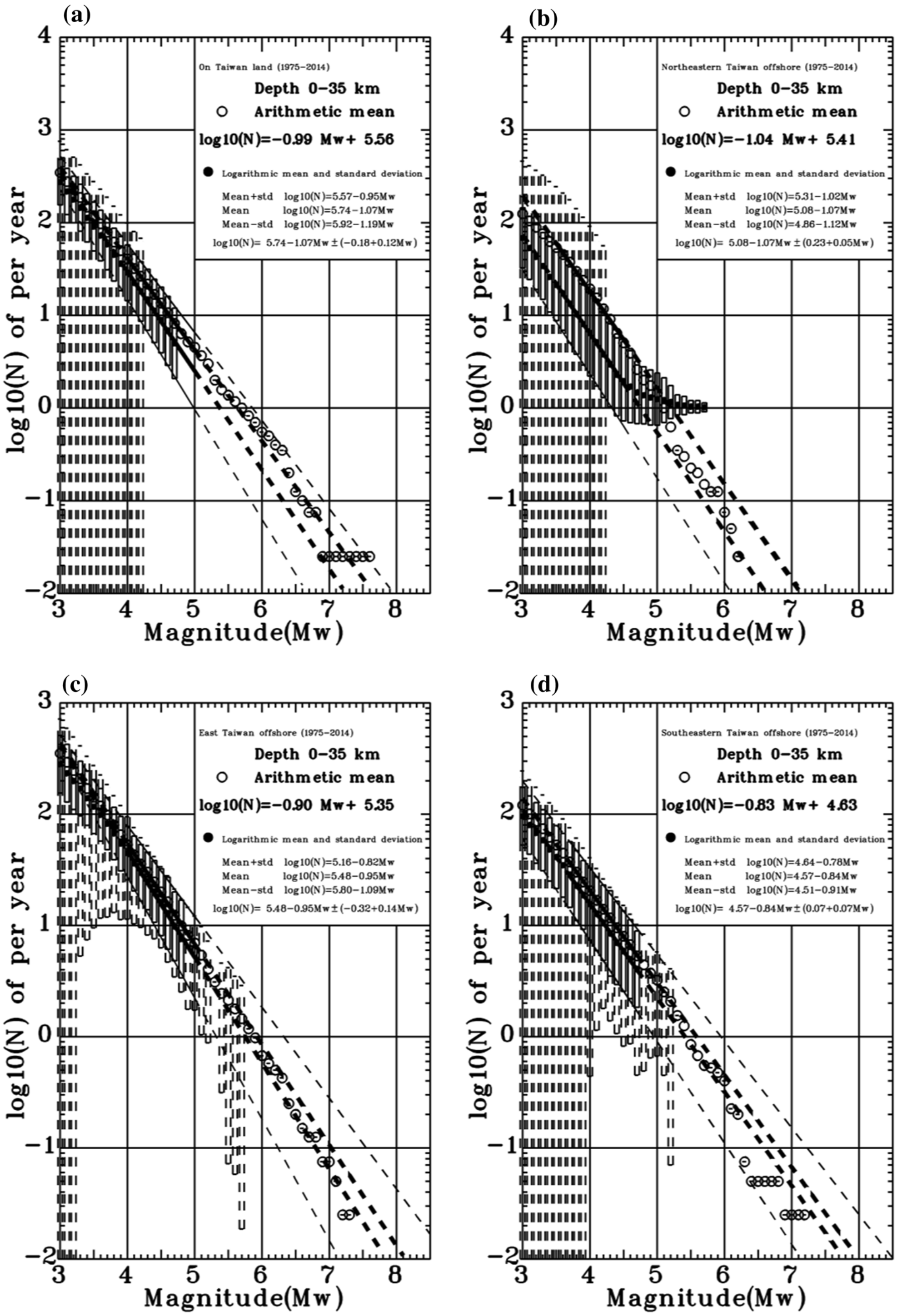
For eastern Taiwan offshore, as shown in Fig. 3c:

$$\begin{aligned}
&\text{Mean} + \text{std} \quad \log_{10} N = 5.16 - 0.82M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.997 \\
&\text{Mean} \quad \log_{10} N = 5.48 - 0.95M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.998 \\
&\text{Mean} - \text{std} \quad \log_{10} N = 5.80 - 1.09M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.997 \\
&\text{Combined} \quad \log_{10} N = 5.48 - 0.95M_w \\
&\quad \pm (-0.32 + 0.14M_w)
\end{aligned} \tag{8}$$

For southeastern Taiwan offshore, as shown in Fig. 3d:

$$\begin{aligned}
&\text{Mean} + \text{std} \quad \log_{10} N = 4.64 - 0.78M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.999 \\
&\text{Mean} \quad \log_{10} N = 4.57 - 0.84M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.999 \\
&\text{Mean} - \text{std} \quad \log_{10} N = 4.51 - 0.91M_w, \\
&\quad \text{fitting coefficient } r^2 = 0.997 \\
&\text{Combined} \quad \log_{10} N = 4.57 - 0.84M_w \\
&\quad \pm (0.07 + 0.07M_w)
\end{aligned} \tag{9}$$

From Fig. 3, we can see that the logarithmic standard deviations are symmetric about their



logarithmic means and increase slightly with magnitude. These equations are very well constrained by the data for $M_w \leq 5.0$, as demonstrated by the near unity fitting coefficient, r^2. This phenomenon is used to obtain robust estimates of the median annual seismicity rate and its upper and lower bounds for

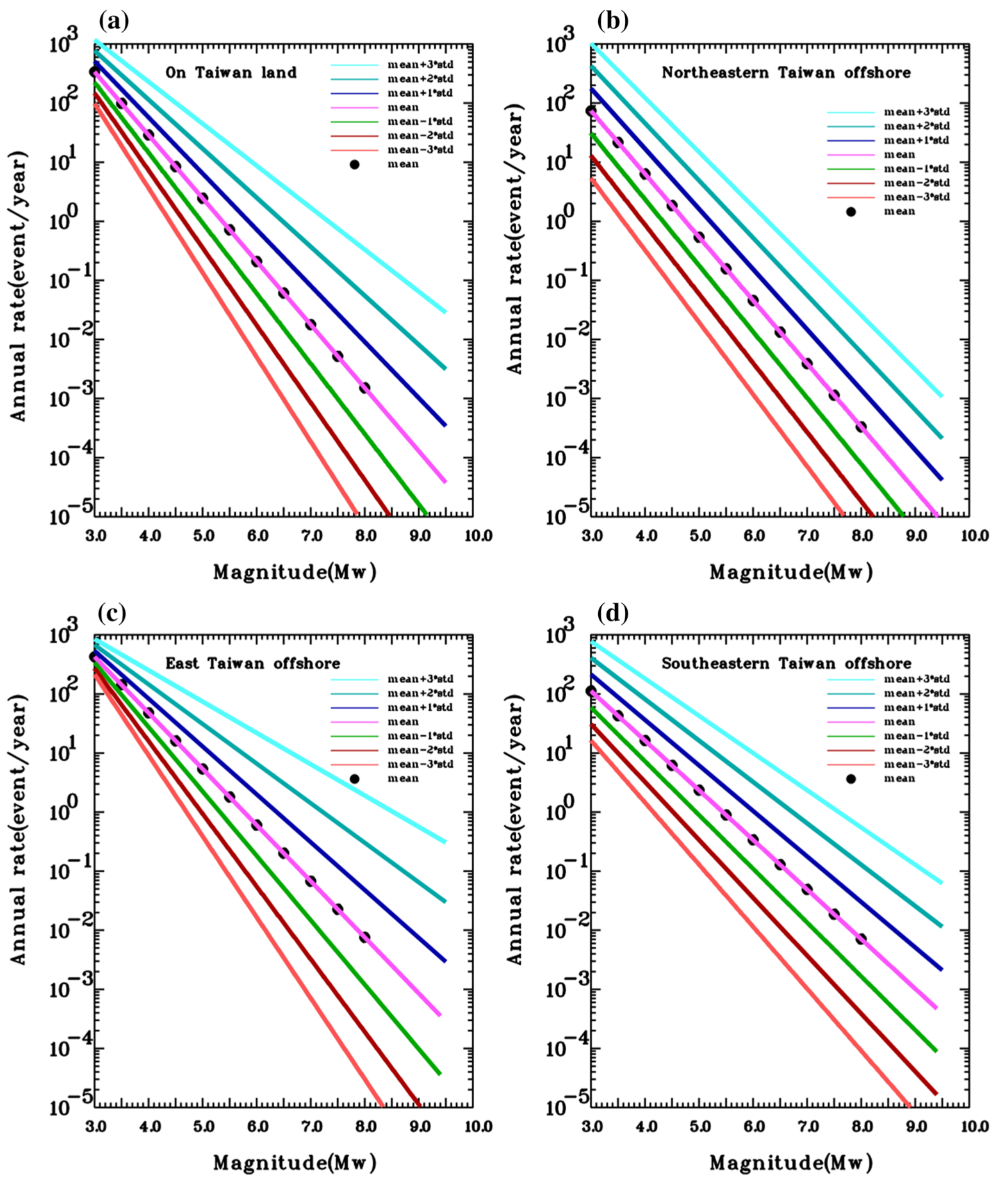


Figure 4

Alternative Gutenberg–Richter relation, including the median annual seismicity rate and its standard deviation for earthquakes of in Taiwan region. **a** Inland Taiwan, **b** northeastern Taiwan offshore, **c** eastern Taiwan offshore, and **d** southeastern Taiwan offshore

$M_w \geq$ 5.0. The figure shows that the observed arithmetic mean annual seismicity rates are almost all enveloped within the logarithmic mean ± one standard deviation lines. This indicates that the logarithmic mean method is more suitable to be applied to the probabilistic seismic hazard analysis than the conventionally expressed arithmetic mean Gutenberg–Richter relation.

5. Forecasting the Probabilities of Large Earthquakes

We begin to develop the innovative forecasting methodology to forecast the probabilities of future larger earthquakes from results above. First, from Eqs. (6)–(9), we can obtain the annual seismicity rate as a function of magnitude (M_w), as shown in Fig. 4

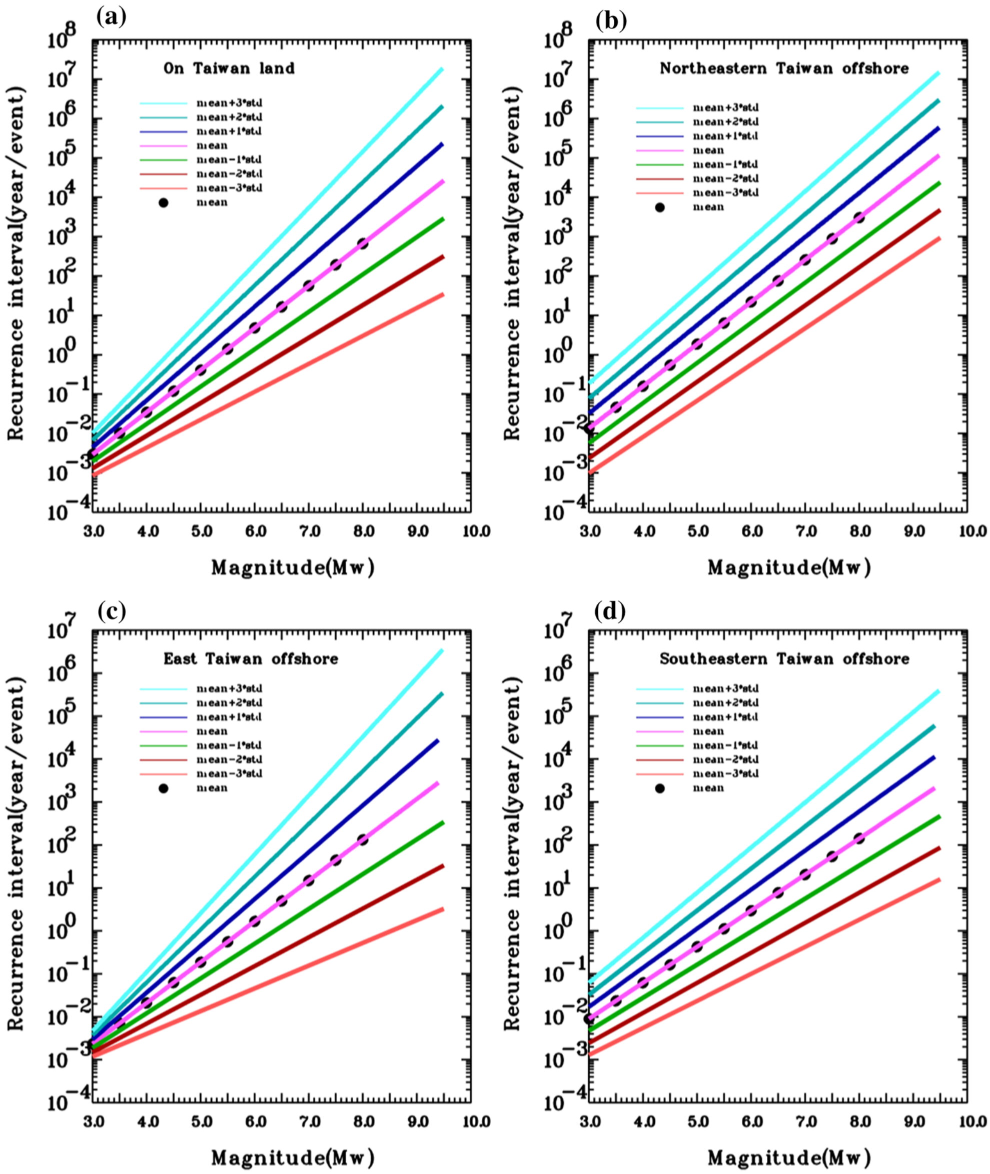


Figure 5

The Log T vs. M relation for the median recurrence interval and its standard deviation as function of M for earthquakes of in Taiwan region, **a** inland Taiwan, **b** northeastern Taiwan offshore, **c** eastern Taiwan offshore, and **d** southeastern Taiwan offshore

for four different regions of Taiwan. Subsequently, we invert the alternative G–R relation to obtain a corresponding relation for the logarithmic mean recurrence interval and its standard deviation; for example, Eq. (6) becomes $\log_{10} T = -5.74 + 1.07M_{\mathrm{w}} \pm (0.18 - 0.12M_{\mathrm{w}})$, and the results are shown in Fig. 5 for four different regions of Taiwan. Since the logarithmic annual seismicity rates and the corresponding recurrence intervals follow closely the lognormal distribution, we can obtain the cumulative probabilities of earthquakes as a function of the recurrence interval. A few discrete numerical results from on Taiwan land data set are given below:

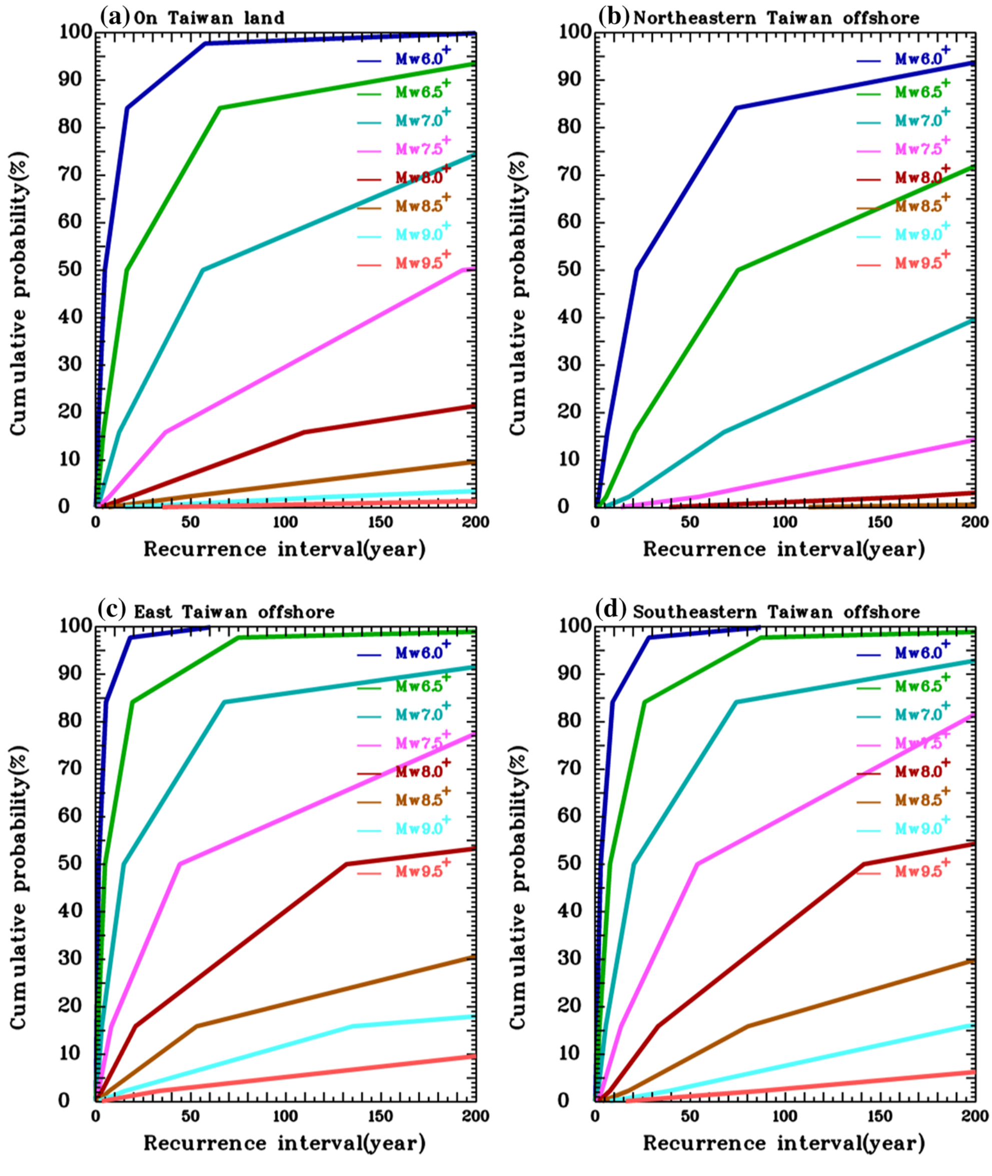


Figure 6
Cumulative probability curve of cumulative probability function as a function of the recurrence interval for M_w 6.0$^+$ to M_w 9.5$^+$ earthquakes in thye Taiwan region, **a** inland Taiwan, **b** northeastern Taiwan offshore, **c** eastern Taiwan offshore, and **d** southeastern Taiwan offshore

M	0.135%	2.275%	15.87%	50.0%	84.13%	97.72%	99.865%
6.0	0.11	0.40	1.38	4.79	16.60	57.54	199.53
7.0	0.59	2.69	12.30	56.23	257.04	1174.90	5370.32
8.0	3.02	18.20	109.65	660.69	3981.08	23988.36	144544.3

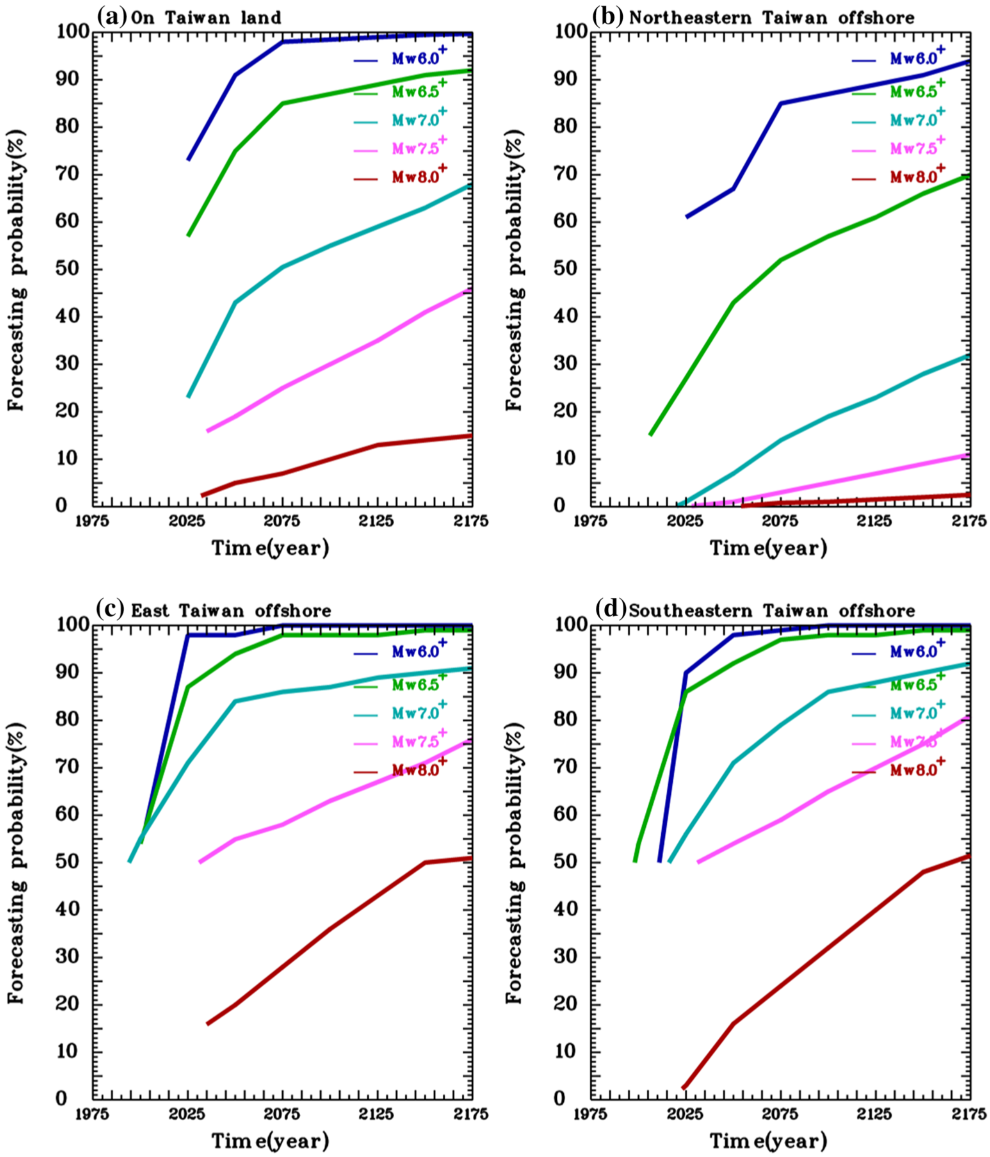


Figure 7
Cumulative probabilities of earthquakes in the Taiwan region since the most recent earthquakes of each given magnitude, as of December 31, 2014, **a** inland Taiwan, **b** northeastern Taiwan offshore, **c** eastern Taiwan offshore, and **d** southeastern Taiwan offshore

The results are shown in Fig. 6a–d for inland Taiwan, northeastern Taiwan offshore, eastern Taiwan offshore, and southeastern Taiwan offshore, respectively. From the above data show that the cumulative probability of an M_w 7.0$^+$ earthquake would increase up to 50% in 56.23 years, and an M_w 8.0$^+$ earthquake would arrive at 15.87% in 109.65 years.

Finally, we can find the probabilities of future earthquakes by picking the date of the most recent earthquake of any given magnitude to start the clock. For example, the dates of the most recent earthquakes inland Taiwan, as of December 31, 2014, are as follows:

M	Year	Month	Day	Date (decimal)
6.0	2013	3	27	2013.236
6.5	1999	9	25	1999.734
7.0	2003	12	10	2003.942
7.5	1999	9	20	1999.720
8.0	2014	12	31	2014.999

We can fix these dates as the starting time of the cumulative probabilities obtained as Fig. 6 to find the cumulative probabilities of future earthquakes at any given time, for example, a few discrete numerical results are given below for inland Taiwan:

M	2025	2050	2075	2100	2125	2150	2175
6.0	73%	91%	98%	98.5%	99%	99.5%	99.7%
6.5	57%	75%	85%	87%	89%	91%	92%
7.0	23%	43%	51%	55%	59%	63%	68%
7.5		19%	25%	30%	35%	41%	46%

The results are shown in Fig. 7. From Fig. 7, we ca see there are lower probabilities of larger earthquakes occurring in northeastern Taiwan offshore than inland Taiwan, eastern Taiwan offshore, and southeastern Taiwan offshore in the future. Also, from Fig. 7a, we can see an M_w 7.0$^+$ earthquake and an M_w 7.5$^+$ earthquake would have a probability of 23% and 15.87% to occur on Taiwan land by 2025 and 2035, respectively.

6. Discussion

It is difficult to accurately predict the occurrence of future large earthquakes; therefore, we use probabilistic approaches to process seismic hazard analysis. Although the earthquake datasets follow the Gutenberg–Richter relation, in order to increase the confidence, we need the mean annual seismicity rate and its standard deviation to quantitatively estimate the probability of future earthquakes.

In this study, we develop an innovative approach to represent the Gutenberg–Richter relation including the logarithmic mean. The characteristic of this approach is that the logarithmic standard deviation, with respect to the corresponding logarithmic mean annual seismicity rate, is symmetric in the log-linear coordinates. Therefore, the logarithmic mean annual seismicity rate explicitly includes the logarithmic standard deviation in the log-linear Gutenberg–Richter relation. However, one shortcoming of our new approach is that the event numbers cannot be zero in an individual region; accordingly, in order to increase the confidence of calculated results, the event numbers must be greater than about 10 in an individual region. The logarithmic mean is found to have a more well-behaved lognormal distribution.

In this study, we propose an innovative forecasting methodology based on characteristics of the logarithmic mean method. Logarithmic mean median annual seismicity rate explicitly incorporates the standard deviation; therefore, we can obtain the median, median ± 1 * standard deviation, median ± 2 * standard deviation, and median ± 3 * standard deviation recurrence interval for any given magnitTnd, the logarithmic mean appears a well-behaved lognormal distribution, the characteristic can provide the cumulative probability of median, median ± 1 * standard deviation, median ± 2 * standard deviation, and median ± 3 * standard deviation, respectively. Combining the two characteristics, we can forecast the cumulative probability of future larger earthquake of any given magnitude and time. These results can be of benefit for preventing and mitigating seismic hazard.

7. Conclusion

This study demonstrates the merit of the logarithmic mean. Our results show that when larger event numbers are removed the logarithmic mean seismicity rate and its standard deviation only decrease slightly. This phenomenon indicates that the logarithmic mean method tends to dampen the influence of surprisingly large or surprisingly low event numbers in an earthquake dataset.

From the forecasting results, we can see that there are lower cumulative probabilities of future larger earthquakes for northeastern Taiwan offshore than inland Taiwan, eastern Taiwan offshore, and southeastern Taiwan offshore, and it has higher probabilities in eastern Taiwan offshore.

As we change the magnitude increment ΔM, the recurrence time of forecasting future large earthquakes will slightly change because of the completeness of our earthquakes dataset, resulting in only a slight change in the Gutenberg–Richter equation, but the difference will increase as forecasting future earthquakes magnitudes increase.

It is worth noting that an M_w 7.0$^+$ earthquake and an M_w 7.5$^+$ earthquake would have probabilities of 23% and 15.87% to occur on inland Taiwan by 2025 and 2035, respectively. It is of benefit for preventing and mitigating seismic hazard.

8. Data and Resources

All data sources are taken from published works listed in the References. Some plots were made using Generic Mapping Tools version 4.3.1 (http://www.soest.hawaii.edu/gmt; Wessel and Smith 1998, last accessed August 2006).

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References

Aki, K. (1965). Maximum likelihood estimate of b in the formula log $N = a - bM$ and its confidence limits. *Bulletin of Earthquake Research Institute, University of Tokyo, 43,* 237–239.

Bayrak, Y., & Bayrak, E. (2012). Regional variations and correlations of Gutenberg–Richter parameters and fractal dimension for the different seismogenic zones in Western Anatolia. *Journal of Asian Earth Sciences, 58,* 98–107.

Chang, W. Y., Chen, K. P., & Tsai, Y. B. (2016). An updated and refined catalog of earthquakes in Taiwan (1900–2014) with homogenized M_w magnitudes. *Earth, Planets and Space, 68*(45), 1–9. https://doi.org/10.1186/s40623-016-0414-4.

Chen, K. P., & Tsai, Y. B. (2008). A catalog of Taiwan earthquakes (1900–2006) with homogenized M_w magnitudes. *Bulletin of the Seismological Society of America, 98,* 483–489. https://doi.org/10.1785/0120070136.

Cheng, S. N. & Yeh, Y. T. (1999) Photo Album of ten disastrous earthquakes in Taiwan. Report Central Weather Bureau and Inst. Earth Sci., Acad. Sin., ROC, 255 pp **(in Chinese)**.

Gutenberg, B., & Richter, C. F. (1941). Seismicity of the earth. *Geological Society of America, Special Papers, 34,* 1–131.

Gutenberg, B., & Richter, C. F. (1944). Frequency of earthquakes in California. *Bulletin of the Seismological Society of America, 34,* 185–188.

Hsu, M. T. (1971). Seismicity of Taiwan and some related problem. *Bulletin of the International Institute of Seismology and Earthquake Engineering, 8,* 41–160.

Kijko, A. (1985). A modified form of the Gutenberg–Richter magnitude-frequency relation: Maximum likelihood estimation of its parameters. *Bulletin of the Seismological Society of America, 75,* 319–322.

Ma, K. F., Lee, C. T., Tsai, Y. B., Shin, T. C., & Mori, J. (1999). The Chi-Chi, Taiwan earthquake: Large sueface displacements on an inland thrust fault. *Eos, Transactions American Geophysical Union, 80,* 605–611.

Ogata, Y., & Zhuang, J. (2006). Space-time ETAS models and an improved extension. *Tectonophysics, 413,* 13–23.

Shin, T. C., & Teng, T. L. (2001). An overview of the Chi–Chi, Taiwan earthquake. *Bulletin of the Seismological Society of America, 91,* 895–913.

Tsai, Y. B. (1985). A study of disastrous earthquakes in Taiwan, 1683–1985. *Bulletin of Institution of the Earth Science, 5,* 1–44.

Wang, J. H., & Kuo, H. C. (1995). A catalogue of Ms > 7 Taiwan earthquakes (1900–1994). *Journal of the Geological Society of China, 38,* 95–106.

Wessel, P., & Smith, W. H. F. (1998). New improved version of generic mapping tools released. *Eos, Transactions American Geophysical Union, 79,* 579.

Wyss, M. (1973). Towards a physical understanding of the earthquake frequency distribution. *Geophysical Journal of the Royal Astronomical Society, 31,* 341–359.

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